Time : 2 hours Max Marks : 40

TOPOLOGY III - MID-SEMESTRAL EXAM.

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

(1) Let X be a space and let $S_*(X)$ denote its singular chain complex. For an abelian group G, define

$$h_n(X;G) = H_n(\mathrm{Hom}(G, S_*(X))$$

where $\mathrm{Hom}(G, S_*(X))$ denotes the chain complex whose n-th chain group is $\mathrm{Hom}(G, S_n(X))$ with he obvious boundary map. Compute the groups $h_n(X;G)$ when $G = \mathbb{Z}, \mathbb{Z}_2, \mathbb{Q}$. [6]

(2) Suppose that the sequence

$$A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

of abelian groups is exact. Show that the dual sequence

$$0 \longrightarrow \mathrm{Hom}(C,G) \xrightarrow{\tilde{g}} \mathrm{Hom}(B,G) \xrightarrow{\tilde{f}} \mathrm{Hom}(A,G)$$

is exact for every abelian group G. Further show that if f is injective and the first sequence splits then $\tilde{f}$ is surjective and the second sequemce splits. [10]

(3) Prove that there does not exist a manifold X such that S^4 is homeomorphic to $X \times S^2$. What is a more general statement that one can make? [6]

(4) Let $X = \mathbb{R}P^2 \times S^3$. Compute the groups $H^i(X;G)$ for $G = \mathbb{Z}, \mathbb{Z}_2$. [10]

(5) Define the notion of orientability of a manifold. Show that a connected orientable manifold has exactly two orientations. [2+6]